Using Brinson Attribution to Explain the Differences Between Time-Weighted (TWR) and Money-Weighted (IRR) Returns

Stock managers and investors have historically used the time-weighted return (TWR) as the sole performance reporting measure because industry standards have endorsed the use, not to mention the fact that equity indices and benchmarks are reported on the same basis. The TWR lends itself nicely to daily priced, daily traded and liquid investments. It's not surprising that bonds are reported in the same manner. As alternative asset classes and newfangled investment structures evolved, including closed-end vehicles in private equity and venture capital, it became clear that the TWR didn't quite fit, although it is still desired by chief investment officers because one return measure is needed to aggregate performance and to compare performance across multiple asset classes.

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The industry standards have been slow to recognize the need for the money-weighted return (IRR), although it appears to be gaining some traction. In recent years, there have been papers written about new measures that mimic the IRR as well as increased discussions in the industry about the importance of it. For example, D'Alessandro (2011) presents a weighted average rate of return that combines the periodic variable rate of return feature of the TWR with the variable dollar weighting feature of the IRR. Also, "Money-Weighted Return" (2015) states, "Could the two approaches (TWR and IRR) complement one another in helping asset owners explain and properly attribute actual economic gains and losses experienced by the plan? And could broader understanding of money-weighted returns lead to their increased adoption?"

Part of the problem is that many are still debating two basic questions; 1) *Should time-weighted or money-weighted returns be reported and 2) Which is a better measure?* The answers are simply both and neither! Both returns are totally different, yet both are absolutely relevant. For example, for private equity real estate closed-end funds, where the manager controls cash flows, the IRR is a good measure of the manager's performance, however, the TWR is also useful as it allows investors a comparison to real estate open-end funds where the manager doesn't control the cash flows, other asset classes like stocks and bonds where the TWR is king, as well as to create time-weighted return indices that are required for asset allocation, portfolio optimization, benchmarking, attribution, and risk analysis. The critics will argue that presenting two different returns will be confusing to investors, especially when one measure shows a positive return and another measure shows a negative return. However, with proper disclosures and reports, both are clearly needed.

In practice today, many simply say that the difference is just the timing of cash flows, where the IRR takes timing and amounts into account and the TWR does not. Up until now, there has been no formal mathematical reconciliation between the two. With the creative use of the existing well-known Brinson attribution model, however, we can finally end the debate and take a critical step forward in improving the consistency and transparency of reporting investment performance results. The focus of this article therefore is to recommend to the industry that both the TWR and the IRR should be reported along with a reconciliation of the two using the *"D'Alessandro TWR - IRR Reconciliation Method."*

Prior to unveiling this new technique, we must first provide a refresher on attribution, a broad term that means many things, and what a TWR and IRR is. During this review, keep in mind the end game - a relatively simple, transparent, mathematical model to explain the difference between the TWR and the IRR.

Attribution comes in many shapes and sizes. Generally speaking, attribution is all about analyzing performance returns to determine what is driving the return or the excess return when compared to something else. There is absolute attribution that looks within to determine, for example, which property-type sectors are contributing most to the overall fund return. This is known as "contribution to return." There is also "relative" attribution that compares the performance of one fund to another, typically a third party benchmark. For relative attribution, however, there are different models available. The arithmetic attribution model simply defines excess return as a manager's fund return less the benchmark return. The geometric attribution model, defines excess return as the manager's fund return divided by the benchmark return (technically it's one plus the manager's return divided by one plus the benchmark return, minus one). With arithmetic attribution, the excess return is compared to the benchmark starting point, whereas with geometric attribution the excess return is compared to the benchmark ending point. Sound confusing? Well, it is at first. A simple example with numbers will surely help.

1. Assumptions
   a. Manager A and Manager B both have $100 to invest
   b. Manager A's return was 30% and the respective benchmark was 20%
   c. Manager B's return was 10% and the respective benchmark was 1%
2. Using arithmetic attribution
   a. Manager A's excess return is (30% - 20%) or 10%
   b. Manager B's excess return is (10% - 1%) or 9%
      i. *Manager A out-performed Manager B*
3. Using geometric attribution
   a. Manager A's excess return is (1+30%) / (1+20%) – 1 or 8.33%
   b. Manager B's excess return is (1+10%) / (1+1%) -1 or 8.91%
      i. *Manager B out-performed Manager A*

As you can see, using the same set of assumptions, Manager A out-performed Manager B using arithmetic attribution, but Manager B out-performed Manager A using geometric attribution. So which attribution model is right? They both are. It just depends on your perspective. As stated earlier, arithmetic compares the excess performance to the benchmark starting point and geometric compares it to the benchmark ending point. Here is what we mean by that;

1. Manager A
   a. Excess return was 10% on $100 or $10
   b. Arithmetic attribution compares this $10 excess return to the benchmark starting point of $100 ($10/$100) for an excess return of 10%
2. Geometric attribution compares this $10 excess return to the benchmark ending point of $120 (the benchmark performed 20%). Therefore the excess return is $10 / $120 or 8.33%

The same math can be applied to Manager B. Arithmetic attribution is typically used in the United States whereas geometric attribution is primarily used in Europe. Some say arithmetic attribution is more intuitive, but others say the geometric model is more proportional. Since arithmetic attribution uses the benchmark starting point, the excess return will be higher than the geometric model in an up market. On the flip side, the excess return using arithmetic attribution will be lower than geometric in a down market.

To complete the refresher on attribution we need to review the Brinson attribution framework. Brinson is a form of relative attribution and was introduced to the investment industry in 1985, known as Brinson-Hood-Beebower (BHB).

The BHB framework evaluates excess performance based on the manager's decision making process. A portfolio manager that has discretion as to which sectors to invest in and which properties within those sectors to buy should be evaluated as to how well he or she executes those decisions. Which sectors to invest in are known as allocation decisions. Which properties to buy are known as selection decisions.

Let's look at two examples. In the first example, the fund and related benchmark returns were 3.05% and 2.25%, respectively, determined as follows;

1. Manager makes the following allocation decisions;
   a. 80% investment in Sector A. Manager's Sector A had a 3.50% return. The contribution to return for Manager's Sector A is, therefore, 80% * 3.50% or 2.80 percent.
   b. 20% investment in Sector B. Sector B had a 1.25% return. The contribution to return for Sector B is, therefore, 20% * 1.25% or 0.25 percent.
   c. Based on the manager's allocation decisions and the related sector performance, the total fund return is 2.80% + 0.25% or 3.05 percent.
2. The Benchmark allocation and sector performance is;
   a. 30% investment in Sector A. Sector A in the benchmark had a 4.00% return. The contribution to return for the benchmark Sector A is, therefore, 30% * 4.0% or 1.20%
   b. 70% investment in Sector B. Sector B had a 1.50% return. The contribution to return for Sector B is, therefore, 70% * 1.50% or 1.05%
   c. Based on the benchmark's allocation and sector performance, the total benchmark return is 1.20% + 1.05% or 2.25%.

How did the manager's fund out-perform the benchmark by 0.80% (3.05% - 2.25%) if it under-performed the benchmark in both sectors? A picture is worth a thousand words, so let's look at the above scenario in graphical form as shown in Figure 1.

Clearly, the answer is the manager did a good job in sector allocation. He or she made an active decision to invest a higher proportion of the fund's investments in Sector A. This 80% allocation was a good choice as Sector A was the better performing sector.

If we look at this using the Brinson attribution model it would look something like Table 1;

The Allocation score is calculated as the difference in weights multiplied by the benchmark return as noted in column G. The overall score of a positive 1.25% indicates that the manager did a good job in deciding how much to invest in each sector. The Selection score is cal-

Figure 1

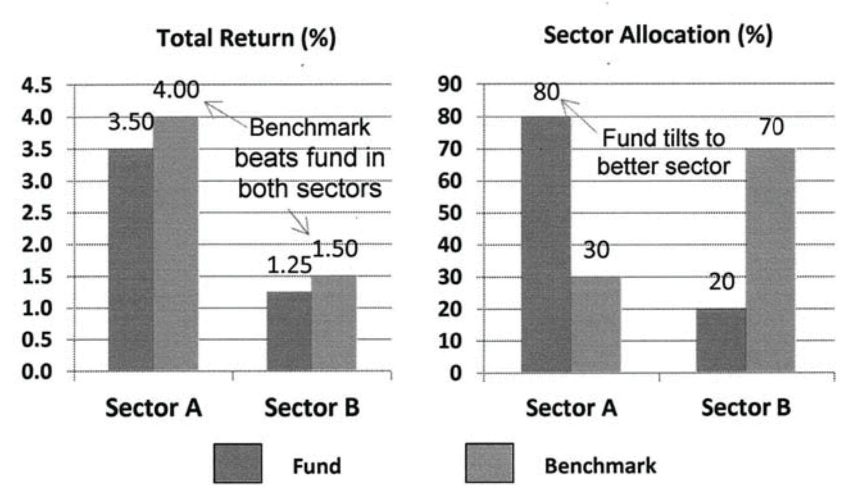


Table 1

A1	B	C	D	E	F	G	H	I	J
2		Fund		Benchmark		Allocation	Selection	Interaction	Excess Return
3		Weight	Return	Weight	Return	(C-E)*F	(D-F)*E	(C-E)*(D-F)	SUM(G:I) or (C*D)-(E*F)
4	Sector A	80%	3.50%	30%	4.00%	2.00%	-0.15%	-0.25%	1.60%
5	Sector B	20%	1.25%	70%	1.50%	-0.75%	-0.18%	0.13%	-0.80%
6		100%	3.05%	100%	2.25%	1.25%	-0.33%	-0.13%	0.80%

culated as the difference in rates multiplied by the benchmark weight as indicated in column H. The overall negative -0.33% score means that the manager did a poor job in selecting individual investments within each sector. The interaction score is a cross product term that results from the interwoven relationship that exists between weights and returns. As indicated in column I, it is the difference in weights multiplied by the difference in returns. Interaction will be positive when the manager either over-weights an overperforming sector or under-weights an underperforming sector. The score will be negative if the manager over-weights an underperforming sector or under-weights an overperforming sector.

With attribution behind us, we now need to focus on time-weighted and money-weighted returns. The current industry standards primarily rely on the TWR for performance reporting. The industry organization that sets the standards for performance reporting worldwide is the CFA Institute, which publishes the Global Investment Performance Standards (GIPS®). The TWR is the primary measure used for reporting and comparing manager performance because it puts all managers on an equal footing. The TWR focuses on the periodic returns earned by each manager rather than the amount of money invested at any given point in time. It doesn't matter if Manager A receives additional money to invest during a period as the timing of such money is weighted or "neutralized" so as not to give such manager an advantage or possibility to earn more of a return for that period compared to another manager who did not get additional money to invest. It doesn't matter if Manager A has $1 in assets under management (AUM) vs. Manager B who has $1 billion in AUM. All that matters is the rate of return earned by each manager.

The IRR is probably more known than the TWR because it is taught in school and is a common function in calculators and spreadsheets. Specifically, the IRR is very sensitive to the timing of capital contributed, the timing of distributions and the amount of money invested at any given point in time. That is why the IRR is sometimes referred to as a "dollar or money-weighted return." Because of the sensitivity to the timing and amount of cash flows, the IRR is typically presented on a since inception basis and for vintage periods to allow managers and investors to appropriately compare portfolios that were created in the same year. The IRR is the discount rate required to make the net present value of all the cash flows equal to zero. The IRR imputes periodic values and assumes re-investment of all distributions which are factored in the overall IRR rate. This is a very different concept from the TWR, which requires periodic actual fair market values and reduces weighted-average equity by distributions since the manager no longer has to earn a return on money leaving the investment.

The IRR is unique from the TWR because it only requires one valuation (a terminal value or reversion) at the end of the measurement period (holding period). So, in a sense, the IRR only has a single period return, commonly known as a since inception return. The IRR, therefore, works well for certain investments, like real estate development or private equity, where periodic "accurate" valuation may not be possible. In contrast, as we noted above, the TWR requires a valuation for every single measurement period during the holding period.

There are two situations where the TWR and the IRR will be equal. The first situation is when there are no subsequent cash flows other than the initial contribution and the final reversion. The second situation is where the return is earned evenly for every measurement period throughout the holding period.

Now that we've reviewed the basic building blocks of attribution and returns, let's do a deep dive to gain a better understanding of how to assemble these components to accomplish our goal to mathematically reconcile the TWR to the IRR, or vice-versa, using the *"D'Alessandro TWR - IRR Reconciliation Method."* For starters, the TWR and the IRR are virtually complete opposite formulas:

1. The IRR takes into account the timing of cash flows. The TWR goes out of its way to neutralize the timing of cash flows.
2. The IRR value weights the overall return to when most of the cash is invested. The TWR equally weights each measurement period's return regardless of how much money is invested.
3. The IRR imputes one rate of return over the entire holding period. The TWR has multiple rates of return for each measurement period. In other words, the re-investment rate varies between the two.

In order to mathematically compare and contrast these opposing formulas, we can creatively use the existing attribution models that similarly compare one result to another. As we discussed earlier, the Brinson attribution models help us evaluate why a portfolio manager outperforms or underperforms the benchmark. In other words, what active decisions or actions did the portfolio manager take to generate alpha. Alpha or excess return

Table 2

A1	B	C	D	E	F	G	H (same as U)	I	J
2	Quarter End	**Begin Bal**	**Contribs**	**Distribs**	**Return$**	**End Bal**	**Return%**	**Return% + 1**	**IRR**
3	12/31/2014	0.00	100.00			100.00			(100.00)
4	3/31/2015	100.00			4.00	104.00	4.00%	1.0400	0.00
5	6/30/2015	104.00			20.80	124.80	20.00%	1.2000	0.00
6	9/30/2015	124.80	20.00		9.98	154.78	8.00%	1.0800	(20.00)
7	12/31/2015	154.78	40.00		9.29	204.07	6.00%	1.0600	(40.00)
8	3/31/2016	204.07			(20.41)	183.66	-10.00%	0.9000	0.00
9	6/30/2016	183.66		(50.00)	(27.55)	106.11	-15.00%	0.8500	50.00
10	9/30/2016	106.11			29.71	135.83	28.00%	1.2800	0.00
11	12/31/2016	135.83			19.02	154.84	14.00%	1.1400	154.84
12	Total	**1,113.26**	**160.00**	(50.00)	**44.84**	**1,168.10**	***6.88%***	***6.01%***	***IRR is 4.08%***
13		*SUM(C3:C11)*	*SUM(D3:D11)*	*SUM(E3:E11)*	*SUM(F3:F11)*	*SUM(G4:G11)*	*Arithmetic Avg*	*GeoMetric Avg*	

Table 3
(refer to Table 2 for cell references)

A1	AG	AH	AI	AJ	AK
2		**TWR**	**IRR**	**Excess**	**Excess Annualized Multiplier**
3	*Quarterly Formula*	*(1+AH5)^(1/4)-1*	*IRR(J3:J11)*	*AH4-AI4*	
4	Quarterly	6.01%	4.08%	***1.93%***	
5	Annualized	26.29%	17.30%	***8.99%***	4.65
6	*Annualized Formula*	*GEOMEAN(I4:I11) ^4-1*	*XIRR(J3:J11,$B $3:$B$11)*	*AH5-AI5*	*AJ5/AJ4*

for this purpose is simply the portfolio manager return minus the benchmark return (arithmetic approach). The decomposition of the excess return into manager sector allocation and manager investment selection decisions is critical. If we extend this model to evaluating the TWR and the IRR, we can get answers to similar questions. Why did the TWR outperform or underperform the IRR? How much of the differential is due to the timing and size of the cash flows in relation to revaluation, or the difference in reinvestment rates? Attribution combines weights and returns, a form of contribution to return, in a way such that we can better understand what is driving the over or under performance of the TWR compared to the IRR, or vice versa.

Let's look at an example with some numbers. In Table 2, we are evaluating a two-year time horizon or holding period, and are using quarterly measurement periods. As shown in Table 2, the investor contributed \$100 on 12/31/14 and made additional investments on 9/30/15 and 12/31/15 of \$20 and \$40, respectively. On 6/30/16, the investor received a \$50 distribution. Each quarter the investor earned a total return as depicted in column F which was re-invested. Over the two-year period, the investor made \$44.84, and at the end of 12/31/16 has an investment balance of \$154.84.

For illustration purposes, we're assuming the TWR is the "manager" portfolio and the IRR is the "benchmark." This can be switched and the overall excess return results will be similar, however, the allocation, selection and interaction scores will vary; a topic for another paper. Based on this cash flow activity and market value changes, the quarterly TWR and IRR were 6.01% and 4.08%, respectively. The TWR outperformed the IRR by 1.93% or 193 basis points on a quarterly basis. On an annualized basis this translates to a 26.29% TWR compared to a 17.30% IRR, or an excess annualized return of 8.99 percent. The formulas used to calculate the TWR and the IRR as well as the annualization of the quarterly returns are depicted in Table 3.

Now for the fun stuff! How do we explain the excess return or outperformance of the TWR over the IRR? In order to answer this question we need to implement the Brinson attribution model.

First, we must calculate the weights and returns for both the TWR and the IRR. As discussed earlier, the TWR equally weights the returns over the time horizon, whereas the IRR value-weights the return. Since there are 8 quarters in the two-year time holding period, the TWR will assign an equal weight each quarter of 12.50% (100% / 8 quarters). For purposes of determining the value weights each period we use the imputed IRR weights as shown in column N in Table 4. These imputed weights are calculated starting with the initial investment and increased each period by the quarterly IRR growth rate, plus contributions less distributions. These values are end of period balances. However. Since returns are based on beginning period balances, adjusted for external cash flows, it would be more accurate to use the beginning balances (for simplicity here, cash flows are assumed to occur at the end of the quarter), as depicted in Column O.

The rates are also shown in Table 4. For the TWR, the reinvestment rate is the actual quarterly return each period which varies from quarter to quarter, adjusted for geometric compounding. This adjustment is necessary to properly calculate the contribution to return for the holding period as demonstrated in Table 5 column V. The adjustment is simply the ratio of the quarterly geometric average to the arithmetic average (6.01% / 6.88% or 87.39%) multiplied by the actual time-weighted return for each quarter. For example, the 3/31/15 actual rate of return of 4.00% is multiplied by 87.39%, resulting in 3.50 percent. The same calculation is applied to each quarter. When you calculate the contribution to return, which is the weight multiplied by the return for each quarter, and then sum the quarterly contribution to returns, you get the overall geometric average of 6.01% as shown in Column X. For the IRR, the reinvestment rate is the actual IRR which is constant every quarter. This dynamic highlights one of the main differences between the TWR and IRR, and is worth repeating. The TWR equally weights variable reinvestment rates, whereas the IRR value weights fixed reinvestment rates.

It should be noted that the *D'Alessandro Time & Money Weighted Return (TMWR)*, mentioned earlier, would be calculated as the total return dollars of \$44.84 in Column F of Table 2, divided by the sum of the weighted average denominators for each quarter of \$1,113.26, or Column C in Table 2, resulting in a 4.03% TMWR. This compares closely to the IRR, which can be calculated in a similar manner. The total return dollars are obviously the same, however, the weights are different. The sum of the weighted average denominators for the IRR is $1,100.39 as shown in Column O of Table 4. When you divide the total return dollars of $44.84 by the $1,100.39, the result is the IRR of 4.08 percent. This highlights two interesting key points. First, the IRR can quickly be estimated without a black box formula, simply by dividing the cumulative profits by the sum of the actual quarterly weighted average equity denominators. Second, and most importantly, it highlights that the TMWR is a more accurate dollar-weighted return measure than the IRR because the TMWR uses actual quarterly weighted average equity, whereas the IRR only imputes or estimates quarterly weighted average equity. Now that we know the weights and returns, the next step is to apply the Brinson attribution model. Table 6 demonstrates this.

The weight impacts are calculated as the difference in weights multiplied by the benchmark return, or in this case the IRR, as shown in column AB. The term EW refers to equal weighted used by the TWR and VW refers to value weighted used by the IRR. The rate im-

Table 4

A1	B	N	O	P (same as W)	Q (same as V)	R	S
2	Quarter End	Investment Balance ($100 grown at quarterly IRR plus Contribs less Distribs)	Average IRR Investment Balance	TWR Investment Equal Weights over time horizon	Quarterly TWR re-investment Rate	IRR Investment Value Weights over time horizon	Quarterly IRR Re-investment rate
3		100.00		100% / # of Qtrs		O / $O$12	
4	3/31/2015	104.08	100.00	12.50%	3.13%	9.09%	4.08%
5	6/30/2015	108.32	104.08	12.50%	19.13%	9.46%	4.08%
6	9/30/2015	132.73	108.32	12.50%	7.13%	9.84%	4.08%
7	12/31/2015	178.14	132.73	12.50%	5.13%	12.06%	4.08%
8	3/31/2016	185.40	178.14	12.50%	-10.87%	16.19%	4.08%
9	6/30/2016	142.95	185.40	12.50%	-15.87%	16.85%	4.08%
10	9/30/2016	148.78	142.95	12.50%	27.13%	12.99%	4.08%
11	12/31/2016	154.84	148.78	12.50%	13.13%	13.52%	4.08%
12	Total	**1,155.23**	**1,100.39**	**100.00%**		**100.00%**	
13		*SUM(N4:N11)*	*SUM(O4:O11)*	*SUM(P4:P11)*		*SUM(R4:R11)*	

Table 5
(refer to Table 4 for references to columns P thru S)

A1	B	U (same as H)	V (same as Q)	W (same as P)	X	Y	Z
2	Quarter End	Return%	Arithmetic to Geometric Conversion	TWR Investment Equal Weights over time horizon	Contribution to TWR (EW*TWR)	Contribution to IRR (VW*IRR)	Excess Return (TWR-IRR)
3			$V$12/$U$12*U		V * W	R * S	X - Y
4	3/31/2015	4.00%	3.50%	12.50%	0.437%	0.37%	0.07%
5	6/30/2015	20.00%	17.48%	12.50%	2.185%	0.39%	1.80%
6	9/30/2015	8.00%	6.99%	12.50%	0.874%	0.40%	0.47%
7	12/31/2015	6.00%	5.24%	12.50%	0.655%	0.49%	0.16%
8	3/31/2016	-10.00%	-8.74%	12.50%	-1.092%	0.66%	-1.75%
9	6/30/2016	-15.00%	-13.11%	12.50%	-1.639%	0.69%	-2.33%
10	9/30/2016	28.00%	24.47%	12.50%	3.059%	0.53%	2.53%
11	12/31/2016	14.00%	12.24%	12.50%	1.529%	0.55%	0.98%
12	Total	**6.88%**	**6.01%**		**6.01%**	**4.08%**	**1.93%**
13		*AVERAGE(U4:U11)*	*AVERAGE(V4:V11)*		*SUM(X4:X11)*	*SUM(Y4:Y11)*	*SUM(Z4:Z11)*

Table 6

A1	B	AB	AC	AD	AE	AF
2	Quarter End	**Weight Impacts (EW-VW)*IRR**	**Rate Impacts (TWR-IRR)*VW**	**Timing Impacts**	**TWR-IRR Spread**	**CHECK**
3		(P-R)*S	(Q-S)*R	(P-R)*(Q-S)	SUM(AB:AD)	AE-Z
4	3/31/2015	0.14%	-0.05%	-0.02%	0.07%	0.00000%
5	6/30/2015	0.12%	1.27%	0.41%	1.80%	0.00000%
6	9/30/2015	0.11%	0.29%	0.08%	0.47%	0.00000%
7	12/31/2015	0.02%	0.14%	0.01%	0.16%	0.00000%
8	3/31/2016	-0.15%	-2.07%	0.47%	-1.75%	0.00000%
9	6/30/2016	-0.18%	-2.90%	0.75%	-2.33%	0.00000%
10	9/30/2016	-0.02%	2.65%	-0.10%	2.53%	0.00000%
11	12/31/2016	-0.04%	1.10%	-0.08%	0.98%	0.00000%
12	Total	**0.00%**	**0.43%**	**1.51%**	**1.93%**	0.00000%
13		*SUM(AB4:AB11)*	*SUM(AC4:AC11)*	*SUM(AD4:AD11)*	*SUM(AE4:AE11)*	

Table 7

A1	AL	AM	AN	AO
2	**Description**	**Quarter**	**Annualized**	**Annualized Formulas**
3	**TWR**	**6.01%**	**26.29%**	*(1+AM3)^4-1*
4	***less rate diff if TWR was value-weighted like IRR***	*0.43%*	*1.98%*	*AM4*$AK$5*
5	***less IRR bad timing***	*1.51%*	*7.01%*	*AM5*$AK$5*
6	**IRR**	**4.08%**	**17.30%**	*SUM(AN3:AN5)*
7	*TWR - IRR "spread"*	*1.93%*	*8.99%*	*AN3-AN6*

pacts are calculated as the difference in rates multiplied by the benchmark weight. The timing impacts are the difference in weights multiplied by the difference in rates. The sum of weight, rate and timing impacts is the TWR-IRR spread or excess return. On a quarterly basis, the TWR out-performed the IRR by 1.93% that consisted of 0.43% due to rate impacts and 1.51% due to timing impacts. Table 7 provides an overall recap of the reconciliation.

Since we solved for the quarterly rates, we must use the multiplier (proportion of the annualized excess return to the quarterly excess return or 8.99%/1.93%), calculated in Table 3 cell AK5 of 4.65, to derive the annualized amounts in column AN. On an annualized basis, therefore, the TWR outperformed the IRR by 8.99% that can be explained as 7.01% due to bad timing in relation to market valuation changes that are not considered by the IRR and 1.98% representing the reinvestment rate dif-

Table 8

A1	B	AQ	AR	AS	AT	AU	AV	AW	AX
2	Quarter End	Allocation score when TWR Over-weights	Allocation score when TWR Under-weights	Selection score when TWR Out-performs	Selection score when TWR Under-performs	IRR Bad timing decisions	IRR Bad timing decisions	IRR Good timing decisions	IRR Good timing decisions
3	12/31/2014	EW>VW	EW<VW	TWR>IRR	TWR<IRR	TWR OW UP	TWR UW UP	TWR OW UP	TWR UW OP
4	3/31/2015	0.14%	0.00%	0.00%	-0.05%	0.00%	0.00%	-0.02%	0.00%
5	6/30/2015	0.12%	0.00%	1.27%	0.00%	0.41%	0.00%	0.00%	0.00%
6	9/30/2015	0.11%	0.00%	0.29%	0.00%	0.08%	0.00%	0.00%	0.00%
7	12/31/2015	0.02%	0.00%	0.14%	0.00%	0.01%	0.00%	0.00%	0.00%
8	3/31/2016	0.00%	-0.15%	0.00%	-2.07%	0.00%	0.47%	0.00%	0.00%
9	6/30/2016	0.00%	-0.18%	0.00%	-2.90%	0.00%	0.75%	0.00%	0.00%
10	9/30/2016	0.00%	-0.02%	2.65%	0.00%	0.00%	0.00%	0.00%	-0.10%
11	12/31/2016	0.00%	-0.04%	1.10%	0.00%	0.00%	0.00%	0.00%	-0.08%
12	Total	**0.39%**	**-0.39%**	**5.45%**	**-5.02%**	**0.49%**	**1.22%**	**-0.02%**	**-0.18%**
13		*Allocation score = .39% + -.39% = .00%*		*Selection score = 5.45% + -5.02% = .43%*		*Interaction score = .49% + 1.22% + -.02% + -.18% = 1.51%*			

Table 9

A1	BC	BD	BE	BF
2				
3	**Quarterly Periods, when**	**Spread**	**% of Total Spread**	**# Qtrs**
4	*TWR>IRR*	5.72%	52%	6
5	*TWR<IRR*	-5.30%	48%	2
6	***less rate diff if TWR was value-weighted like IRR***	**0.43%**	**22%**	8
7	*bad timing*	1.71%	89%	5
8	*good timing*	-0.20%	11%	3
9	***less IRR bad timing***	**1.51%**	**78%**	8
10	***TWR - IRR "spread"***	**1.93%**		

ferential if the TWR was value-weighted like the IRR. We can provide further details of the weight, rate, and timing impacts to provide more insight into the differences. In essence, we are grouping the impacts calculated in Table 6 into categories that are relevant for the impact being measured. For instance, for the weight impacts, we can aggregate the quarters when the TWR was over-weighted compared to the IRR (Table 8 column AQ) and separately we can aggregate the quarters when the TWR was under-weighted compared to the IRR (Table 8 column AR). With regard to rate impacts, we can aggregate the quarters when the TWR over-performed the IRR (Table 8 column AS) and separately the quarters when the TWR under-performed the IRR (Table 8 column AT). With regard to timing impacts, which is critical for capturing the manager's investment timing decisions; we can aggregate the scores into four important categories. One when the TWR overweights and over-performs (Table 8 column AU), two when the TWR underweights and underperforms (Exhibit #8 column AV) (both being poor IRR timing), three when the TWR overweights and underperforms (Table 8 column AW), and four when the TWR under-weights and over-performs (Table 8 column AX) (the latter two being good IRR timing).

With any combinations of returns and cash flows, the weight impact differences will always equal zero. Even though the TWR and the IRR weights differ each quarter, the average weight over the holding period will always be the same. If you average either the IRR imputed balances in Table 4 column N or the beginning IRR balances in column O, you get 12.50%, the same as 1/8 for the TWR. Since the holding period average weight for both the TWR and IRR is equal, the allocation score will always be 0. Even though both return formulas are weighted differently, one being equally weighted the other value-weighted, the average weight over time for both is exactly the same!

With this additional detail we could present further insights into the TWR-IRR spread or excess return as depicted in Table 9. For example, we can see that for six out of eight quarters the TWR overperformed the IRR. More importantly, for five out of eight quarters the IRR shows bad timing, which was the primary driver of the IRR underperforming the TWR.

CONCLUSION

What does this all mean and how might it actually be used in practice? First, those using the Brinson attribution model might explore analyzing and reporting "interaction" as depicted here rather than burying it in either Allocation or Selection. Second, as we mentioned earlier, the investment community should stop debating which return measure to report. Third, the industry standards should require both the TWR and the IRR (even better, the TMWR) to be reported, along with an accompanying reconciliation of the two as depicted by the *"D'Alessandro TWR - IRR Reconciliation Method."* The sample report shown as Appendix #1 could be presented

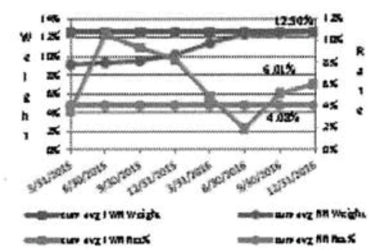
Appendix

D'Alessandro TWR - IRR Reconciliation Method

TWR - IRR Reconciliation

Two years ended 12/31/2016

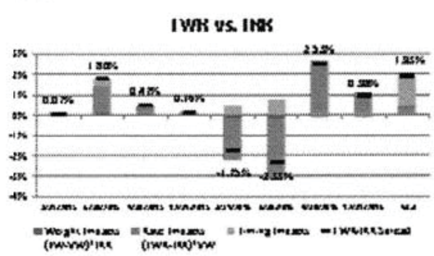
Summary		
Description	Quarter	Annualized
TWR	6.01%	26.29%
[illegible]	0.48%	1.98%
[illegible]	1.51%	7.01%
IRR	4.08%	17.30%
TWR-IRR "spread"	1.93%	8.99%



The cumulative average quarterly weight of the TWR is constant vs. the IRR weight which varies but converges to the TWR. The cumulative average quarterly return of the TWR varies vs. the IRR which is constant over the two year period.

Details of the TWR - IRR spread			
Quarterly Periods, when	Spread	% of Total Spread (absolute values)	# Qtrs
TWR>IRR	3.72%	52%	6
TWR<IRR	-3.30%	48%	2
[illegible]	0.48%	22%	8
bad timing	1.71%	89%	5
good timing	-0.20%	11%	3
[illegible]	1.51%	78%	8
TWR - IRR "spread"	1.98%		

Weight, Rate and Timing impacts vary each quarter. Over the two year period, the Weight impacts "zero out", leaving the TWR-IRR Spread explained solely by Rate and Timing impacts



Quarterly Data									D'Alessandro TWR - IRR Reconciliation Method			
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	IRR	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
12/31/14	0.00	100.00	0.00	100.00			-100.00					
03/31/15	100.00	0.00	4.00	104.00	4.00%	1.0400	0.00	100.00	0.14%	-0.05%	-0.02%	0.07%
06/30/15	104.00	0.00	20.80	124.80	20.00%	1.2000	0.00	104.00	0.12%	1.27%	0.41%	1.80%
09/30/15	124.80	20.00	9.98	134.78	8.00%	1.0800	-20.00	124.80	0.11%	0.29%	0.08%	0.47%
12/31/15	134.78	40.00	9.29	204.07	6.00%	1.0600	-40.00	134.78	0.02%	0.14%	0.01%	0.16%
03/31/16	204.07	0.00	-20.41	183.66	-10.00%	0.9000	0.00	204.07	-0.15%	-2.07%	0.47%	-1.75%
06/30/16	183.66	-50.00	-27.55	106.11	-15.00%	0.8500	50.00	183.66	-0.12%	-2.90%	0.75%	-2.33%
09/30/16	106.11	0.00	22.71	153.35	20.00%	1.2000	0.00	106.11	-0.02%	2.65%	-0.10%	2.58%
12/31/16	153.35	0.00	19.02	134.84	14.00%	1.1400	134.84	153.35	-0.04%	1.10%	-0.08%	0.98%
Total		110.00	44.84		Arith Avg 6.66%	Compound Avg 6.01%	IRR is 4.08%		0.00%	0.48%	1.51%	1.98%

Prepared by Joe D'Alessandro   CONFIDENTIAL DO NOT DISTRIBUTE   7/22/2016 to investors that compares and reconciles the two returns in a numerical as well as graphical format. This report would help users better understand the drivers of investment performance and the differences between two commonly used return metrics. If you have any questions for Mr. D'Alessandro, please contact him at JoeD@realestateinsights.com.

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