

# The “Missing Link” in Benchmarking Private Equity Performance and a New Twist on “Alpha”

*Over the past twenty plus years, various quantitative methodologies have been developed to facilitate comparisons between private equity fund internal rates of return (PEF IRR), and public market benchmark time-weighted rates of return (TWR). Long and Nickels (1996) got the private equity “benchmarking ball” rolling when they introduced the Index Comparison Method, later renamed Public Market Equivalents (LN-PME). This method invests the private equity fund’s investor cash flows in the public market benchmark to calculate a public market equivalent IRR (PME IRR). The over or underperformance (IRR spread) is the PEF IRR minus the PME IRR. Unfortunately, potential disparities in private and public market performance present challenges for the LN-PME method. Since then, there have been proposed methods to correct for the private and public market performance differences; however, they use heuristic approaches and have additional mathematical difficulties. Two promising methods were introduced in 2014. Gredil, Griffiths, and Stucke’s (2014) Direct Alpha method extended the Kaplan and Schoar (2005) private market multiple method. Global Endowment Management’s (2014) GEM IPP method expanded Philappaou and Gottschalg’s (2005) excess IRR method. Both Direct Alpha and GEM IPP calculate the IRR spread using a “benchmark return plus a market premium” model. All methods assume the IRR spread is arithmetic and each produces different results. Until now, no method has been capable of identifying the implied cash flows and the related PME IRR. As a result, it is impossible to know which method is correct or the most accurate. This article solves the dilemma, provides a solution and the missing link for private equity benchmarking. It is now possible to identify the correct IRR spread as well as the public market equivalent portfolio implied cash flows and the related PME IRR.*

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Over the past twenty plus years, various quantitative methodologies have been developed to facilitate comparisons between private equity fund (PEF) performance and an index or a benchmark (sometimes referred to as the reference or replicating portfolio). Such reference portfolios typically are based upon publicly available indices or benchmarks (*i.e.*, public market equivalent); however, for certain asset classes and vehicle structures, such as open-end core real estate, private market indices and benchmarks also can be utilized,

with the chosen index or benchmark typically dependent upon the investment style of the PEF under analysis.<sup>1</sup> For this reason, as used herein, the acronym PME is more broadly defined than usual as either a public or private market equivalent.

The reasons for multiple methods of comparing private to public market (or private to private market) performance is simple – we are dealing with complex math compounded by the unique nature of privately-held

closed-end vehicle investments. Private equity relies extensively on the internal rate of return (IRR) as a reporting metric, whereas public equity and open-end core real estate use the time-weighted rate of return (TWR). Since the IRR considers the timing of cash flows and private equity managers have control of investor capital calls and distributions, the IRR is a better measure of manager performance. In contrast, public equity and open-end real estate fund managers do not control the timing of investor cash flows, therefore, the TWR, which neutralizes the timing of cash flows, is a better measure of manager performance.<sup>2</sup> Because IRR and TWR metrics are vastly different, comparing the two proves immensely challenging and frequently yields limited useful results. The need to compare the two is clear and present — investors, investment managers and consultants must evaluate whether private equity investments performed better, the same or worse than alternative investment choices.

## BACKGROUND

Long and Nickels (1996) developed the Long-Nickels Index Comparison Method, or Public Market Equivalent (LN-ICM or LN-PME), which is often credited as the first method to formally evaluate private equity performance. In this method, the private equity cash flows (*i.e.*, investor contributions and distributions) are assumed to be invested in a PME using the exact timing and amounts of the PEF cash flows under analysis. The only difference in the cash flow streams between the PEF and the PME is the ending net asset value (NAV), which for the PME is the terminal value of the assumed cash flows as if they were invested in the PME. The PEF IRR is then compared to the PME IRR and the difference is the over or underperformance (or the IRR Spread) of the PEF relative to the PME. While this initially sounds very intuitive and easy to implement, LN-ICM has certain challenges when performance significantly diverges between the PEF and PME. For example, strong positive performance by the PEF may result in large distributions that, when applied to the PME during a period of weak performance, could result in large negative market values or short positions in the PME, thereby rendering the assumed PME returns unreliable for comparative purposes. Conversely, the opposite may happen in a market scenario where very weak PEF performance is mismatched against strong PME performance, resulting in large long positions in the PME in later years. Despite

the potential mismatch under certain market conditions, LN-ICM got the “benchmarking ball” rolling for the private equity industry due to the method’s intuitiveness and ease of implementation and, in fact, is still being used today.<sup>3</sup>

Rouvinez and Capital Dynamics (2003) developed the PME plus (PME+) method. PME+ attempts to rectify the potential for short and long positions in the PME under LN-ICM by adjusting the assumed distributions by a fixed scaling factor so that the ending NAV of the PME is the same as the PEF. Unfortunately, making adjustments to the interim cash flows in order to force the ending NAV to be the same between the PME and PEF has unintended impacts on the IRR, making comparisons even more suspect.

Kaplan and Schoar (2005) created the KS-PME method, which advanced the choices through the introduction of a market multiple similar to the total value to paid in (capital) multiple (TVPI) but using compounded future values (or discounted present values) instead of the actual cash flows.<sup>4</sup> Specifically, KS-PME future values the distributions and ending NAV; then divides such by the future value of the contributions. The future values of the contributions and distributions are based on the periodic returns of the PME. The KS-PME multiple then is compared to the PEF TVPI and the difference, or multiplier, is the over or underperformance. The benefits of this method are that it is cash flow-based, uses a well-known metric in the private equity industry, and is derived from the Modern Portfolio Theory (MPT) model for expected returns, which consists of both market and non-market return contributions. The disadvantages of KS-PME are that the multiple is neither a return nor return spread and cannot be used in MPT modeling, absent other significant assumptions.

Phalippou and Gottschalg (2005) wrote a paper describing “alpha” or “excess IRR” that was then described in more detail by Global Endowment Management (2014) as GEM Implied Private Premium (GEM IPP). The GEM IPP is calculated by solving for the IRR Spread or “market premium” that when added to the benchmark returns provides the required rate of return such that the future value of the cash distributions equals the future value of the cash contributions. This is similar to an IRR calculation that discounts cash flows to a net present value of zero, or compounds cash flows to a net future

value of zero. Stated another way, under the GEM IPP method, when the net present or future value of cash flows equals zero, the TVPI based on those values will equal one, resulting in an indifference point between the PEF and PME. The drawback of this method is that the PME portfolio cannot be formalized or “replicated.” Therefore, the PME IRR can only be implied as the difference between the PEF IRR and the GEM IPP.

Cambridge Associates (2013) created a method called modified PME (mPME). mPME is similar to PME+, but instead of using a fixed scaling factor for all distributions, it uses a time-varying scaling factor based on interim private equity NAVs. This method has challenges not only from the alteration of cash flows as in PME+, but also introduces another potential bias relating to unreliable pricing of interim private equity NAVs.

The Direct Alpha (DA) method was created by Gredil, Griffiths and Stucke (2014). DA is deduced from KS-PME by computing the future value (or present value) of cash flows and calculating an IRR based on those values. The result is the IRR Spread of the relative performance of the PEF to the PME. The drawback of this method, similar to GEM IPP, is that the PME portfolio cash flows cannot be replicated. Therefore, the PME IRR can only be implied as the difference between the PEF IRR and the DA.

Altshuler and Magni (2015) developed the Aggregate Return on Investment (AROI) method to calculate the return spread. In this method, the PEF cash flows are the same as the PME except for the ending NAV. The difference between the PEF and PME NAVs represents the dollar profit (or loss), depending upon whether the PEF’s NAV is greater than (or less than) the PME’s ending NAV. The difference, referred to as the dollar excess or value-added profit, is then divided by the aggregate periodic market values of the PME to calculate the return spread. The underlying returns are not IRRs, but rather another dollar-weighted return measure. The method’s non-reliance on an IRR is both an advantage and disadvantage. The advantage of not using an IRR is that an IRR may have multiple solutions, especially when cash flows frequently fluctuate between positive and negative values over the holding period. Conversely, the disadvantage of not using the IRR is that the alternate dollar-weighted return metric of the PEF does not agree to the PEF’s actual reported IRR. The return

spread is, therefore, not IRR based. Last, it uses the aggregate benchmark portfolio’s periodic market values (not the PEF’s NAVs) as the denominator rather than a “neutral” denominator, which may create a bias in the results.

In addition to the methods referenced herein, there are many other methods that are less well-known, such as Alignment Capital’s Alternative Index Comparison Method (AICM), Sorenson, Wang and Yang’s Adjusted PME, and Bison-PME. Interestingly, some methods are patented and some are subject to licensing arrangements, so user beware. As new methods continue to emerge, the more confusing the landscape of options will become. Which method should be used under what circumstances? Which method is the most robust mathematically in its derivation and reconciliation of the PEF IRR, the PME IRR and the corresponding IRR Spread?

## THE MISSING LINK

The earlier methods—including LN-PME, PME+ and mPME—attempt to estimate the IRR Spread using a heuristic approach. Because these methods have certain limitations, some of which are summarized above, they are not ideal for evaluating relative performance. Further, the resulting IRR Spread expressed in these methods (as the PEF IRR minus the PME IRR) is not mathematically robust due to the IRR’s complex polynomial compounding equation, and the non-additive properties of IRRs, as discussed in the Gredil, Griffiths, Stucke (2014) paper. While theoretically sound, KS-PME, as mentioned previously, it is not computed in the form of a return but rather a multiple. The Altshuler and Magni method does not utilize the actual PEF IRR and the denominator used to calculate the return spread relies solely on the market values of the PME.

For additional consideration, that leaves us with the later methods of DA and GEM IPP. These two methods are based on models that derive investment performance as a “return plus a market premium,” similar to the capital asset pricing model used in MPT, which states the expected rate of return is equal to a risk-free investment return plus a risk-adjusted market premium. In simplified terms, investors should require a private market premium over the public market return because of the inherent risks of illiquidity and valuation challenges associated with private market investments. Having nar-

rowed the methods down to the DA and GEM IPP, the remaining focus of this article, will illustrate how these two methods work and how they are related.

However, before diving deeper into those two methods, a quick refresher on attribution may be warranted, as it will come into play in the comparison of the two methods. As a first step, relative attribution analysis involves calculating an over or underperformance of a portfolio versus a benchmark. There are two very different approaches in determining the over or underperformance of a portfolio return compared to a benchmark return. One approach is to calculate the arithmetic difference of the portfolio return minus the benchmark return. For example, if the portfolio return was 10% and the benchmark return was 2%, the portfolio overperformed by 8% (10%-2%). This approach to attribution answers the simple question as to how much better (or worse) the portfolio performed in comparison to the benchmark. In other words, we are evaluating two separate and independent investments. But is that the only way to look at it? Of course not! Another approach is to ask the question, how much better did the portfolio perform if an investor actually invested in the benchmark to the same degree? The answer to that question is 7.8%, and it is based upon the geometric difference. The performance difference is calculated as one plus the portfolio return divided by one plus the benchmark return minus one, or, in the case of the example,  $[(1+10\%)/(1+2\%)]-1$  which equals 7.8 percent. The over or underperformance in arithmetic attribution is determined based on the benchmark *starting* point whereas in geometric attribution, the over or underperformance is based on the benchmark *ending* point. What are the implications of this? In a rising market, the arithmetic approach will result in a higher return spread, whereas in a declining market the geometric approach will provide the higher return spread. As a matter of good practice and full transparency, investment firms should disclose how the over or underperformance is defined and calculated.<sup>5</sup> Investors should also consider the implications of how spreads are calculated with regards to performance based fee structures.

Once the over or underperformance is determined, the second step in relative attribution is to determine why the portfolio over or underperformed. Was it due to manager allocation skills, manager selection skills or possibly a combination of the two? The second step in

relative attribution for private equity performance will be a topic for future articles.

Now that we reviewed the basics of calculating over and/or underperformance for relative attribution, we can address the main question: Which of the two PME methods, that is, DA or GEM IPP, should be used to evaluate private equity performance?

As noted, both of these methods use the “benchmark return plus a market premium” model to tackle the problem and, in this regard, they are very similar. However, as we will see shortly, the two methods produce different results.

As shown in Exhibit 1, the PEFs’ actual cash flows (as provided in the Gredil, Griffiths, and Stucke (2014) paper) are in columns C, D, E and F. The PEF IRR is calculated on the net cash flows in column F, which sum to \$250, the profit or the amount by which distributions, including the ending NAV, exceeds contributions. The 17.53% IRR is the annualized return for the nine-year holding period. The public market actual time-weighted annual returns and related index values are shown in columns G and H, respectively. The hypothetical cash flows of DA are shown in columns I through L. These cash flows are the compounded future values of the actual PEF’s cash flows at the PME actual TWRs. For example, the \$131 PME future value contribution is calculated by compounding the actual PEF contribution of \$100 at the public market actual TWR for the nine years (*i.e.*, 12/31/2001 to 12/31/2010). The nine-year cumulative return can be calculated either by geometrically linking 1 + the annual returns in column G or by dividing the ending index value of 131 by the beginning index value of 100. Column L represents the future value of the PME net cash flows. However, since the future values are associated with the actual dates of each respective cash flow, the PME growth rate is neutralized. For example, the \$50 contribution on 12/31/2005 grows at a rate of 11.97% ( $131/117-1$ ), resulting in a future value of \$56. Even though this is a compounded future value, it is still reflected as occurring on 12/31/2005. Since the IRR is sensitive to the timing of cash flows, the compounding is neutralized and because the compounding is based on the actual public market index rate, the return or growth rate of the public market index was also neutralized. The IRR of the future value cash flows of 12.57% therefore represents the IRR

**Exhibit 1: Gredil, Griffiths, and Stucke (2014) Data Used in DA**

A	B	C	D	E	F	G	H	I	J	K	I
63	<b>Direct Alpha</b>	Private Market Actual Values				Public Market		Direct Alpha Cash Flows			
64	Year-end	Contribs	Distribs	NAV	Net Cash Flows	Returns	Index	Contribs Future Value	Distribs Future Value	NAV Future Value	Net Cash Flows Future Value
65	12/31/2001	100	-	-	-100		100	131	-	-	-131
66	12/31/2002	-	-	-	0	-22.00%	78	-	-	-	0
67	12/31/2003	100	25	-	-75	28.21%	100	131	33	-	-98
68	12/31/2004	-	-	-	0	11.00%	111	-	-	-	0
69	12/31/2005	50	150	-	100	5.41%	117	56	168	-	112
70	12/31/2006	-	-	-	0	15.38%	135	-	-	-	0
71	12/31/2007	-	150	-	150	5.19%	142	-	138	-	138
72	12/31/2008	-	-	-	0	-36.62%	90	-	-	-	0
73	12/31/2009	-	100	-	100	25.56%	113	-	116	-	116
74	12/31/2010	-	-	75	75	15.93%	131	-	-	75	75
75	<b>Sum</b>	250	425	75	250			318	455	75	212
76											
77				IRR	<b>17.53%</b>					Direct Alpha	<b>12.57%</b>
78				=IRR(F65:F74,10%)						=IRR(L65:L74,10%)	

Spread or DA between the PEF and the PME. Stated differently, the future valued cash flows represent the actual private equity cash flows plus the benchmark rate of return on those cash flows. The resulting IRR of the future valued cash flows therefore represents the required private market premium or DA. It should be noted that this analysis provides the same results if discounted present values are used instead of compounded future values.

Gredil, Griffiths, and Stucke (2014) do not discuss methodologies for validation of the end result or the calculation of the PME IRR reference portfolio. We know that the PEF IRR is 17.53% and the DA is 12.57%, but what is the PME IRR? The authors state that DA is arithmetic, implying that the PME IRR is 4.96% (17.53% - 12.57%). But is that correct? How can it be tested?

Hold that thought. Let's look at GEM IPP next. GEM IPP represents the implied private market premium that is required above the public market return. GEM IPP future values the cash flows at the public market return plus the private market premium. How do you do that if you don't know what the private market premium is?

Sounds like a circular argument! That is where Excel's goal seek function comes into play. Essentially, to solve, a percentage number (*i.e.*, the Guess Rate) is referenced in a formula that future values the cash flows by the benchmark rate plus the Guess Rate. Then, the sum of the future value of the distributions is divided by the sum of the future value of the contributions to calculate the future TVPI. With Excel's goal seek function, three parameters are set which identify: 1) the TVPI formula cell, 2) the desired value of the TVPI cell, and 3) the Guess Rate cell. Excel then will run multiple iterations to solve for the proper Guess Rate such that the future values of all the cash flows result in a TVPI equal to one. This certainly sounds a lot like the IRR, as Excel solves for the rate at which the net present value (or net future value) of distributions minus contributions equals zero (or in GEM IPP's case, distributions divided by contributions equals one).

Now that the mechanics of DA and GEM IPP are explained, how do they relate to each other, as they both appear to use the concept of solving for the IRR Spread by future valuing cash flows at the benchmark rate plus a private market premium? Let's start with DA. Know-

**Exhibit 2: Future Values the PEF Cash Flows at the DA Rate Calculated in Exhibit 1**

A	B	C	D	E	F	G	H	I	J	K	L
63	<b>Direct Alpha</b>	Private Market Actual Values				Public Market Actual Values		Hypothetical Public Market Portfolio (PME)			
64	Year-end	Contribs	Distribs	NAV	Net Cash Flows	Returns	Index	Contribs Future Value	Distribs Future Value	NAV Future Value	Net Cash Flows Future Value
65	12/31/2001	100	-	-	-100		100	290	-	-	-290.26
66	12/31/2002	-	-	-	0	-22.00%	78	-	-	-	0.00
67	12/31/2003	100	25	-	-75	28.21%	100	229	57	-	-171.79
68	12/31/2004	-	-	-	0	11.00%	111	-	-	-	0.00
69	12/31/2005	50	150	-	100	5.41%	117	90	271	-	180.76
70	12/31/2006	-	-	-	0	15.38%	135	-	-	-	0.00
71	12/31/2007	-	150	-	150	5.19%	142	-	214	-	213.97
72	12/31/2008	-	-	-	0	-36.62%	90	-	-	-	0.00
73	12/31/2009	-	100	-	100	25.56%	113	-	113	-	112.57
74	12/31/2010	-	-	75	75	15.93%	131	-	-	75	75.00
75	<i>Sum</i>	250	425	75	250			610	655	75	120
76											
77				IRR	<b>17.53%</b>					PME IRR	<b>4.41%</b>
78				=IRR(F65:F74,10%)						=IRR(L65:L74,10%)	

ing the IRR Spread calculated by the DA method, is it possible to solve for the actual PME IRR? Instead of future valuing the cash flows at the actual PME benchmark TWR to determine the IRR Spread, what would happen if the cash flows are future valued at the known IRR Spread, specifically the DA rate of 12.57 percent?

Exhibit 2 is the same as Exhibit 1 except for columns I, J, K and L. In Exhibit 1, the cash flows were compounded at the benchmark PME TWR. In Exhibit 2, the future value compounding of the cash flows is calculated using the DA derived rate of 12.57 percent. The resulting net cash flows are shown in column L. Because the PEF cash flows in the hypothetical public market portfolio were compounded at the DA rate, with the dates remaining unchanged, the IRR Spread between the PEF and DA is neutralized. Therefore, the IRR of 4.41% represents the calculated PME IRR, which differs from the arithmetic difference of 4.96% noted previously as surmised by Gredil, Griffiths, and Stucke (2014). Which one reflects the correct PME IRR? How can we reconcile these two different answers?

Remember our relative attribution refresher? The geometric difference represents the return spread on a proportional basis as if the investor invested in the benchmark. Such is shown by calculating one plus the PEF IRR divided by one plus DA minus one or  $[(1+17.53\%)/(1+12.57\%)]-1 = 4.41\%$ , which is the PME IRR calculated in Exhibit 2. Therefore, DA is not an arithmetic difference but a geometric difference!

Is this correct? Can we triple check the results? Let's revisit GEM IPP. As stated earlier, GEM IPP solves for the required return spread by future valuing the cash flows at the public market rate of return plus the private market premium such that the sum of the future net cash flows is equal to zero, or the TVPI on a future value cash flow basis is equal to one.

Exhibit 3 is the same as Exhibits 1 and 2 except for columns I through M. Columns I through K are the future values of the cash flows grown at the benchmark public market equivalent rate of return plus the implied private premium or GEM IPP with column L being the

**Exhibit 3: Depicts the Results for GEM IPP Method**

A	B	C	D	E	F	G	H	I	J	K	L	M	
132	<b>GEM IPP</b>	Private Market Actual Values				Public Market		IPP Cash Flows					
					Net Cash Flows	Returns	Index	Contribs Future Value	Distribs Future Value	NAV	Net Cash Flows Future Value	Years Left	
133	Year-end	Contribs	Distribs	NAV									
134	12/31/2001	100	0	0	-100		100	385	-		-385	9	
135	12/31/2002	0	0	0	0	-22.00%	78	-	-		0	8	
136	12/31/2003	100	25	0	-75	28.21%	100	301	75		-226	7	
137	12/31/2004	0	0	0	0	11.00%	111	-	-		0	6	
138	12/31/2005	50	150	0	100	5.41%	117	102	307		205	5	
139	12/31/2006	0	0	0	0	15.38%	135	-	-		0	4	
140	12/31/2007	0	150	0	150	5.19%	142	-	202		202	3	
141	12/31/2008	0	0	0	0	-36.62%	90	-	-		0	2	
142	12/31/2009	0	100	0	100	25.56%	113	-	129		129	1	
143	12/31/2010	0	0	75	75	15.93%	131	-	-	75	75	-	
144	<b>Sum</b>	250	425	75	250			789	714	75	0		
145													
146				IRR	17.53%					IPP	13.12%	Guess Rate	
147				=IRR(F65:F74,10%)						TVPI	1.00		
148									TVPI	=(J144+K144)/L144			
149	Example FV Contribs	385	=C134*(((H143/H134)^(1/M134)+I146))^(M134)										

summation of the net cash flows at those future values. As an example, cell I134 is based upon a formula used to derive the future value of the 12/31/2001 \$100 cash flow contribution. For illustrative purposes that formula is depicted in row 149. In the formula shown in row 149, cell C134 reflects the 12/31/2001 \$100 contribution. Also within the formula, the expression (H143/H134) ^ (1/M134) represents the annualized return of the benchmark PME portfolio, which is calculated by dividing the ending index value in cell H143 by the beginning index value in cell H134 (i.e., 131/100 in the example). The result of the annualized return calculation is raised to the power of one divided by the appropriate number of years for which the contribution was in the fund (i.e., for the 12/31/2001 contribution, the correct number of years is nine as shown in cell M134). The “+I146” within the formula is the IPP, which is added to the annualized benchmark return. Lastly, the expression “^M134” compounds the return over the appropriate number of years. This formula is applied to all the cash flows for the respective time periods for which the cash flows are in the benchmark portfolio as determined by the years noted in column M. The TVPI formula depicted in cell K148 is in cell L147, and shows a result of 1.0 after activating the goal seek function in which

Excel ran many iterations to solve for a TVPI of 1 by changing the Guess Rate in cell L146. The Guess Rate that results in a TVPI of 1 is 13.12%, as shown in cell L146. This is the IPP.

By comparison, the DA method calculated the IRR Spread as 12.57% versus the GEM IPP calculation of 13.12 percent. The same question arises: which answer is correct? The answer: they both are! Here’s why. As illustrated in the performance attribution refresher earlier, the DA method provides the geometric difference. The PEF IRR was 17.53 percent. Solving for the PME IRR was performed in two ways: first by future valuing the cash flows at the DA rate, and then calculating the IRR on those future cash flows which resulted in a 4.41% PME IRR, and second, by using the geometric formula of [(1+17.53%)/(1+12.57%)]-1. Because the PEF IRR and the PME IRR are known, subtracting the two results in the IRR Spread on an arithmetic basis or 13.12% (17.53% - 4.41%). This is the same value that the GEM IPP method calculates as the IRR Spread, as shown in Exhibit 3. The GEM IPP method, therefore, represents the arithmetic difference, as stated by the authors. Intuitively and mathematically, this makes sense as the IRR Spread is added to the benchmark return

using the GEM IPP method. Since GEM IPP does not use the IRR function, the result is arithmetic, not geometric. Solving for the PME IRR is simply the PEF IRR minus the IRR Spread or 17.53% - 13.12%, which equals 4.41 percent. Both the DA and GEM IPP methodologies therefore reconcile to the same PME benchmark portfolio return.

Similar to how the PME IRR of 4.41% was validated using the IRR-based DA method, the PME IRR using the GEM IPP method can also be proved as depicted in Exhibit 4.

In Exhibit 4, the cash flows are compounded at the IPP (*i.e.*, the known variable with a value of 13.12% as calculated in Exhibit 3) plus the PME return (the unknown variable as the Guess Rate). For example, as shown in row 149E, the \$100 contribution on 12/31/2001 is compounded for nine years at the GEM IPP of 13.12% plus the PME IRR of 4.41%, which is the guess rate that results in a TVPI of one. Therefore, both the GEM IPP and DA methodologies independently calculate the PME IRR as 4.41 percent.

Although both methods reconcile to the same PME IRR, the DA methodology calculates the IRR Spread geomet-

rically, whereas the GEM IPP methodology calculates the IRR Spread arithmetically. In the GEM IPP paper, the authors demonstrate that DA is a special case of GEM IPP and that under continuous compounding, GEM IPP is equivalent to DA. As proven here, both methods will produce the same PME IRR. The DA methodology has an advantage as it uses an IRR function, which for private equity is the industry standard. A disadvantage with the DA methodology is that the difference is geometric (which some may find confusing or counter intuitive), although the arithmetic difference can simply be derived after the PME IRR is calculated. An advantage with the GEM IPP methodology is that the IRR Spread is arithmetic and easy to explain; however, the calculation is somewhat cumbersome.

One last important observation: Gredil, Griffiths, and Stucke (2014) make the connection between KS-PME and DA by identifying the market multiple as the ratio of the PEF TVPI divided by the KS-PME. In Exhibit 1, the PEF TVPI is 2.00 and is calculated as (D75+E75)/C75 or 500/250. Using the same Exhibit 1, the KS-PME is 1.67 and is calculated as (J75+K75)/I75 or 530/318. Therefore, the market multiple is 1.20, calculated as 2.00/1.67. Because the PME cash flows are now known (as identified in Exhibit 2), the PME TVPI

Exhibit 4: Future Values the Cash Flows at the GEM IPP Plus the PME IRR												
A	B	C	D	E	F	G	H	I	J	K	I	M
132	<b>GEM IPP</b>	Private Market Actual Values				Public Market		PME Cash Flows				
					Net Cash Flows	Returns	Index	Contribs Future Value	Distribs Future Value	NAV	Net Cash Flows Future Value	Years Left
133	Year-end	Contribs	Distribs	NAV								
134	12/31/2001	100	0	0	-100		100	428	-		-385	9
135	12/31/2002	0	0	0	0	-22.00%	78	-	-		0	8
136	12/31/2003	100	25	0	-75	28.21%	100	310	77		-226	7
137	12/31/2004	0	0	0	0	11.00%	111	-	-		0	6
138	12/31/2005	50	150	0	100	5.41%	117	112	336		205	5
139	12/31/2006	0	0	0	0	15.38%	135	-	-		0	4
140	12/31/2007	0	150	0	150	5.19%	142	-	244		202	3
141	12/31/2008	0	0	0	0	-36.62%	90	-	-		0	2
142	12/31/2009	0	100	0	100	25.56%	113	-	118		129	1
143	12/31/2010	0	0	75	75	15.93%	131	-	75	75	75	-
144	<b>Sum</b>	250	425	75	250			850	850	75	0	
145												
146					IRR	17.53%				PME IRR	4.41%	Guess Rate
147					=IRR(F65:F74,10%)					TVPI	1.00	
148										TVPI	= (J144+K144)/L144	
149	Example FV Contribs	428	=C134*(((1+13.12%+L146))^(M134))									

or market multiple of 1.20 is validated as  $(J75+K75)/I75$  or  $730/610$ .

## CONCLUSION

The authors of DA and GEM IPP methodologies both calculate the IRR Spread directly, however, neither appears to have identified the cash flows for the PME reference portfolio in order to calculate the proper PME IRR. In other words, until now, we were not confident which IRR Spread was correct or what the related PME IRR was. By extending the future value concept at differing rates in both models, the fog is lifted and clarity appears. In review, the simple four-step process to prove the results using the DA model is as follows:

- 1) Future value the actual PEF cash flows by compounding such cash flows at the PME benchmark TWR.
- 2) Use the IRR function on the future valued cash flows to calculate the DA or IRR Spread, which is a geometric difference.
- 3) Future value the actual PEF cash flows by compounding such cash flows at the IRR Spread calculated in step 2 above.
- 4) Use the IRR function on the future valued cash flows in step 3 to calculate the true PME IRR.

When it comes to benchmarking a PEF to a public or private market index that uses a time-weighted return metric, choose either the DA or GEM IPP methodology and properly disclose the PME IRR and the related over or underperformance as either arithmetic or geometric.

## A NEW TWIST ON ALPHA

The title of this article includes the words “A New Twist on Alpha.” It actually has a double meaning and provides two unique opportunities: one for private market benchmarking and the other for public market benchmarking.

Although DA uses the term Alpha, it really only calculates the return spread and not “excess return” (ER). In finance, ER implies a difference above a risk-free rate. According to Wikipedia, Alpha, in the context of the

capital asset pricing model (CAPM), was first used in 1968 by Michael Jensen in the evaluation of mutual fund managers. Jensen’s Alpha (“JA”) is a risk-adjusted ER metric, which measures the difference between the actual portfolio or fund return and the portfolio or fund return expected from the CAPM. The formula for JA is:

$$\text{Jensen's Alpha} = \text{Portfolio Return} - [\text{Risk Free Rate} + \text{Portfolio Beta} * (\text{Expected Market Return} - \text{Risk Free Rate})]$$

The first opportunity therefore pertains to benchmarking the private markets, and represents a twist on DA and GEM IPP. Both methods could potentially be risk adjusted by reducing the future value compounding rate, or present value discounting rate, by an appropriate risk-free rate. Further refinement also could be attained by adjusting the compounding or discounting rate for the PEF’s beta relative to the reference benchmark. Such would take benchmarking private market performance to the next level and into the league of the public markets.

The second opportunity pertains to benchmarking the public markets. This would provide a new twist on Alpha, specifically Jensen’s Alpha, which has been used for approximately 50 years in the public markets. This new perspective is simply the geometric version of JA, and the equation is as follows:

$$\text{Geometric version of Jensen's Alpha} = [(1 + \text{Portfolio Return}) / (1 + \text{Risk-free Rate})] / \{1 + [(1 + \text{Expected Market Return}) / (1 + \text{Risk-free Rate}) - 1] * \text{Portfolio Beta}\} - 1$$

In equation format, the geometric version of Jensen’s Alpha is;

$$\alpha_G = \left[ \frac{1 + R_p}{1 + R_f} \right] \div \left\{ 1 + \left[ \frac{1 + R_b}{1 + R_f} - 1 \right] * \beta_p \right\} - 1$$

Where;

$$\begin{aligned} \alpha_G &= \text{Geometric version of Jensen's Alpha} \\ R_p &= \text{Portfolio Return} \\ R_b &= \text{Expected Market Return} \\ R_f &= \text{Risk - free rate} \\ \beta_p &= \text{Portfolio Beta} \end{aligned}$$

If we keep the geometric concept between the portfolio and benchmark rates (Expected Market Return), but ap-

plied an arithmetic concept to the risk-free rate, a variation to the above is:

$$\alpha_{G2} = \frac{1 + (R_p - R_f)}{1 + (R_b - R_f) * \beta_p} - 1$$

As stated earlier, in a rising market, the geometric version of alpha will be lower than the arithmetic version, whereas in a declining market it will be higher. Which method are you using in your portfolio benchmarking toolkit? Does your performance-based fee agreement use geometric or arithmetic math?

Hopefully the topics addressed in this article will provide much food for thought and encourage investment professionals, in both the private and public markets, to explore new ways in analyzing and evaluating investment performance.

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### ENDNOTES

<sup>1</sup> For example, the institutional real estate industry's flagship open-end benchmark, the National Council of Real Estate Investment Fiduciaries Open-end Diversified Core Equity Index or NFI-ODCE, serves as a Private Market Equivalent for core real estate.

<sup>2</sup> For further discussion as to the characteristics of IRR versus TWR, see D'Alessandro's article "A New Measure for the Investment Management Industry: Time- & Money-Weighted Return (TMWR)," *The Journal of Performance Measurement*, Summer 2011.

<sup>3</sup> LN-ICM renamed Public Market Equivalent by Venture Economics in 1996.

<sup>4</sup> TVPI measures the ratio of the current value of remaining investments within a PEF, plus the total value of all distributions to date, relative to the total amount of capital paid into the PEF to date. Pursuant to current GIPS standards, any recallable distributions should be included in the numerator and any reinvested capital that results from recallable distributions should be included in the denominator.

<sup>5</sup> For a further discussion on performance attribution, see D'Alessandro, "Using Brinson Attribution to Explain the Difference Between Time-Weighted (TWR) and Money-Weighted (IRR) Returns," *The Journal of Performance Measurement*, Summer 2016.